Short Papers

Twofold Mur's First-Order ABC in the FDTD Method

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***Abstract*— In this paper, it is shown that the reflection error of Mur's first-order absorbing boundary condition (ABC) can be canceled effectively by applying the ABC twice to an electromagnetic (EM) field on two diagonally neighboring nodes on the x–t, y–t, and z–t planes. Following this idea, we have developed a twofold Mur's first-order ABC (TMFABC), which is efficient to absorb both propagative and evanescent EM waves and very convenient for implementation to multilayered structures. TMFABC improves Mur's first-order ABC more effectively at lower frequencies. This is very important because most energy of a high-speed pulse is concentrated at lower frequencies.**

***Index Terms*—Absorbing boundary condition, FDTD.**

I. Introduction

Solving Maxwell's equations in the time domain is a critical step in various electromagnetic (EM) problems such as scattering, propagation, modeling of microwave components, and high-speed interconnects of integrated circuits. In the past decade, the most commonly used method for the solution is Yee's finite-difference time-domain (FDTD) method [1]. For EM problems in open space, artificial absorbing boundary conditions (ABC's) have to be introduced to truncate the boundless computation domain. With the traditional ABC's such as Mur's ABC's of one-way approximation of a wave equation [2], an accurate solution is available only when the artificial boundary is put sufficiently far away from the scatterer. Recently, two new ABC's have been introduced. One is the dispersive boundary condition [3], [4] based on Higdon's ABC [5]. Another is the perfectly matched layer (PML) [6], which seems to be the most powerful ABC to absorb propagative waves so far.

Myr's first- and second-order ABC's have been widely used in various applications in the past ten years because these ABC's are well built and very convenient for implementation. However, both Mur's first- and second-order ABC's are not satisfactory for absorbing obliquely incident and evanescent waves. Thus, some work has been done to improve the absorption efficiency [7], [8]. In the super-absorbing algorithm [7], an ABC is separately applied to the E- and H-fields at two neighbor nodes on the boundary, and an error-cancellation procedure is then used to reduce the reflection. In [8], Mur's second-order ABC is optimized by choosing the best values for the adjustable parameters in the ABC. Generally speaking, Mur's second-order ABC is more efficient in absorbing propagative waves, but Mur's first-order ABC has been used more frequently in practical applications because it is much easier for implementation and needs no special treatment at corner nodes and for multilayered structures. In this paper, we have developed a twofold Mur's first-order ABC (TMFABC) based on an assumption that the reflection coefficients of an EM field due to an imperfect ABC are the same on two diagonally neighbor nodes on the x–t, y–t, and z–t planes. In TMFABC, we apply Mur's first-order ABC twice to an E-field on two diagonally neighbor nodes such as $(i\Delta x, j\Delta y, k\Delta z, m\Delta t)$ and $((i-1)\Delta x, j\Delta y, k\Delta z, (m-1)\Delta t)$ on the x–t plane. Then the reflection errors are canceled based on this assumption. The idea of error-cancellation in TMFABC is similar with that in the super-absorbing algorithm in [7], but TMFABC has several advantages. First, TMFABC is more simple and convenient for implementation in three-dimensional (3-D) problems. Second, TMFABC can be applied at all nodes of the entire FDTD mesh while the super-absorbing algorithm fails at some special nodes near the mesh corners in 3-D problems. Last, we have found that the absorption efficiency of TMFABC is better than that of the super-absorbing algorithm of Mur's first-order ABC when applied to 3-D microstrips, and that TMFABC is also efficient to absorb evanescent waves. An application example of microstrip line analysis is given to demonstrate the performance improvement of TMFABC for absorbing EM waves. The example shows that TMFABC improves Mur's first-order ABC more effectively at lower frequencies, which is very important because most energy of a high-speed pulse is concentrated at lower frequencies. The voltage definition by linear integration of the electrical field is recommended for modeling high-speed interconnects when the TEM model is no longer satisfactory.

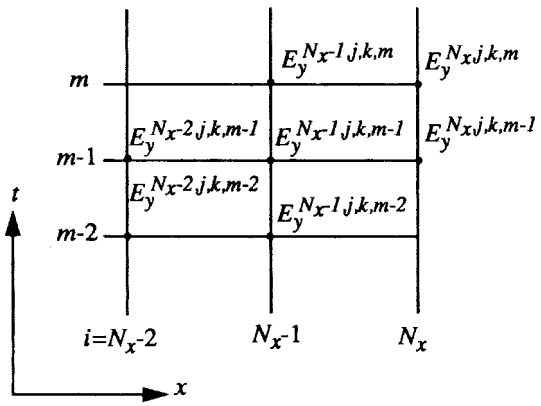

Fig. 1. Field nodes of E_y near the boundary $x = N_x\Delta x$.

II. Twofold Mur's First-Order ABC

The TMFABC developed in this paper is based on an assumption that the reflection coefficients of an EM field due to an imperfect ABC are the same at two diagonally neighbor nodes on the x–t, y–t, and z–t planes. Several nodes of E-field E_y near the boundary at $x = N_x\Delta x$ on the x–t plane are shown in Fig. 1. Let $E_y^{i,j,k,m(1)}$ denote the E_y at $x = i\Delta x$, $y = j\Delta y$, $z = k\Delta z$, and $t = m\Delta t$ obtained by Yee's FDTD algorithm, $E_y^{i,j,k,m(2)}$ denote the E_y at the same node obtained by an ABC, and then from the assumption we have

$$\frac{E_y^{N_x,j,k,m(1)}}{E_y^{N_x,j,k,m(2)}} = \frac{E_y^{N_x-1,j,k,m-1(1)}}{E_y^{N_x-1,j,k,m-1(2)}} = \beta. \tag{1}$$

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When Mur's first-order ABC is considered, β in (1) can be obtained from known E_y at inner nodes, shown in (2) at the bottom of this page, where

$$\rho = \frac{c\Delta t - \Delta x}{c\Delta t + \Delta x} \tag{3}$$

in which $c = c_0/\epsilon_{\text{eff}}^{0.5}$ is the propagation velocity of EM waves which is adjustable in Mur's first-order ABC, where ϵ_{eff} is the effective dielectric constant and c_0 is the velocity of light in vacuum. Thus, the value of $E_y^{N_x, j, k, m(1)}$ on the boundary can be calculated by

$$\begin{aligned} E_y^{N_x, h, k, m(1)} &= \beta E_y^{N_x, j, k, m(2)} \\ &= \beta(E_y^{N_x-1, j, k, m-1(1)} + \rho E_y^{N_x-1, j, k, m(1)} \\ &\quad - \rho E_y^{N_x, j, k, m-1(1)}) \end{aligned} \tag{4}$$

in which $E_y^{N_x, j, k, m(2)}$ denotes the $E_y^{N_x, j, k, m}$ from Mur's first-order ABC. Equation (4) is just the expression of TMFABC in this paper for E_y at the boundary of $x = N_x\Delta x$. The expressions of TMFABC for other EM fields at other boundaries can be derived in a similar way.

For any useful ABC, β should have a value of about one. Our experience with various applications suggests that $\beta <= 1.02$ (which means that when β is found to be larger than 1.02, β is then set to 1.02) in practical use of TMFABC in order to guarantee its stability. We have found that if the absolute value of β is allowed to be larger than 1.5, TMFABC may sometimes be instable. This is because the denominator of (2) contains transient field values and sometimes can be very small (especially at the early period of FDTD simulation when the physical wavefront has not arrived at the boundary), so β may have an extremely large absolute value, which would produce increasingly large oscillation of the computed EM fields from (4).

Like in the super-absorbing algorithm [7], the basic idea in TMFABC to improve an ABC is error cancellation by using the ABC twice. However, the procedure of error cancellation in TMFABC is much simpler than that in the super Mur's first-order ABC, especially for 3-D problems. On the other hand, from (2), (4), and Fig. 1, we can see that TMFABC can be applied at any node of the FDTD mesh. This is true even for inhomogeneous structures such as multilayered microstrip lines.

The absorption efficiency of TMFABC depends on the reliability of the assumption on which TMFABC is based. Suppose $\Psi^{i, j, k(1)}(\omega)$ and $\Psi^{i, j, k(2)}(\omega)$ are the EM field at $x = i\Delta x$, $y = j\Delta y$, and $z = k\Delta z$ in the frequency domain obtained by Fourier transform of the FDTD simulation result and by Fourier transform of the result from an ABC, respectively, $R^{i, j, k}(\omega)$ is the reflection coefficient of the ABC at angular frequency ω, where we have

$$\frac{\Psi^{i, j, k(2)}(\omega)}{\Psi^{i, j, k(1)}(\omega)} = 1 - R^{i, j, k}(\omega) \tag{5}$$

$$\frac{\Psi^{i+1, j, k(2)}(\omega)}{\Psi^{i+1, j, k(1)(\omega)}} = 1 - R^{i+1, j, k}(\omega). \tag{6}$$

We know that the reflection coefficient of Mur's first-order ABC is $(1 - \cos(\theta))/(1 + \cos(\theta))$, where θ is the incident angle of the EM waves to be absorbed. Usually, the space-step Δx in FDTD is much shorter than the wavelength even at the highest frequency, so the incident angle θ at spatial points $(i\Delta x, j\Delta y, k\Delta z)$ and $((i+1)\Delta x, j\Delta y, k\Delta z)$ can be considered approximately the same, and the right-hand sides of (5) and (6) are also the same, which are denoted as $Q(\omega)$ below. From (5) and (6) we can get

$$\Psi^{i, j, k(2)}(t) = q(t)^* \Psi^{i, j, k(1)}(t) \tag{7}$$

$$\Psi^{i+1, j, k(2)}(t) = q(t)^* \Psi^{i+1, j, k(1)}(t) \tag{8}$$

where "*" denotes a convolution, $q(t)$ is the inverse Fourier transform of $Q(\omega)$, and the other time-domain terms in (7) and (8) are also the inverse Fourier transforms of the relevant frequency-domain terms in (5) and (6). If the dispersive property of EM wave propagation is not obvious, $Q(\omega)$ is nearly independent of ω and $q(t)$ can be approximated by a delta function $\delta(t)$. Then, from (7) and (8) we have

$$\frac{\Psi^{i, j, k(2)}(t)}{\Psi^{i, j, k(1)}(t)} = \frac{\Psi^{i+1, j, k(2)}(t)}{\Psi^{i+1, j, k(1)}(t)}. \tag{9}$$

Note that (1) of the assumption is different from (9). However, if (9) is a good approximation, (1) will be a better one because in the FDTD algorithm, the value of an EM field at node $(i-1, j, k, m-1)$ is generally closer to the value at (i, j, k, m) than that at $(i-1, j, k, m)$ if the EM wave propagates in the $+x$-direction. For other kinds of ABC's, the assumption (1) is also found to be reasonable, but not discussed in detail in this paper. From the above discussion we can also conclude that TMFABC improves Mur's first-order ABC more effectively at lower frequencies. This is very important because most energy of a high-speed pulse is concentrated at lower frequencies. The reason TMFABC is more efficient at lower frequencies is that at lower frequencies the dispersive property of EM wave propagation is less obvious and the assumption (1) on which TMFABC is based is more reasonable.

III. Application Example

In this section, an application example of microstrip line analysis is given to illustrate the absorption efficiency of TMFABC. The strip width is 0.15 mm, dielectric thickness is 0.1 mm, and dielectric constant is 13.0. In the FDTD solution, we set $\Delta x = \Delta y = 0.025$ mm, $\Delta z = 0.05$ mm (the z-axis is along the longitudinal direction of the microstrip line) and $\Delta t = 0.5\Delta x/c_0$, and set $0 =< \beta <= 1.02$ for TMFABC. The excitation signal is a Gaussian pulse. The microstrip line is so long that the reflective waves from the far-end boundary do not reach the near-end boundary before the rear tail of sampled EM fields vanishes. On the side boundaries and the far-end boundary, Mur's first-order ABC, TMFABC, and super Mur's first-order ABC are separately used, but on the top boundary near which the EM fields are mostly evanescent, only TMFABC is used because other ABC's cannot efficiently absorb evanescent waves. The average reflection coefficients of the voltage (defined as the integration of E_x under the metal strip center) due to these ABC's in the frequency range of $0 \sim 200$ GHz are shown in Fig. 2 as functions of the effective dielectric constant (ϵ_{eff}). It is recommended to use the linear electrical-field integration definition instead of the power–current definition for the voltage when modeling microstrip lines used as high-speed interconnects in integrated circuits for three reasons. The first reason is that the voltage definition for interconnects must be the same as that for lumped circuit elements. Certainly the linear electrical-field integration definition gives more physical insight about the behavior of lumped circuit elements such as MOS. The second reason is that for multiconductor interconnects, it is difficult

$$\beta = \frac{E_y^{N_x-1, j, k, m-1(1)}}{E_y^{N_x-2, j, k, m-2(1)} + \rho(E_y^{N_x-2, j, k, m-1(1)} - E_y^{N_x-1, j, k, m-2(1)})} \tag{2}$$

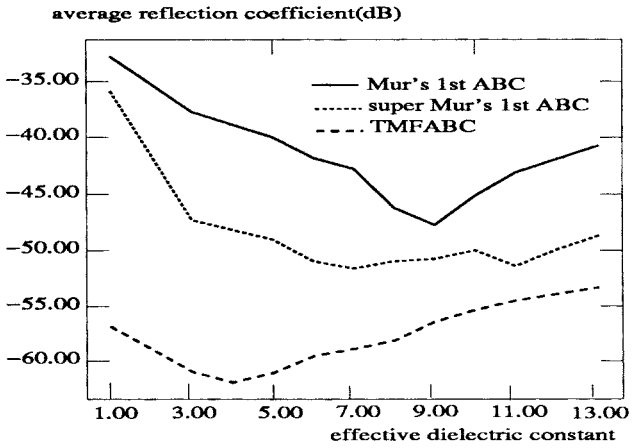

Fig. 2. Average reflection coefficients of three ABC's as functions of the adjustable effective dielectric constant in the example.

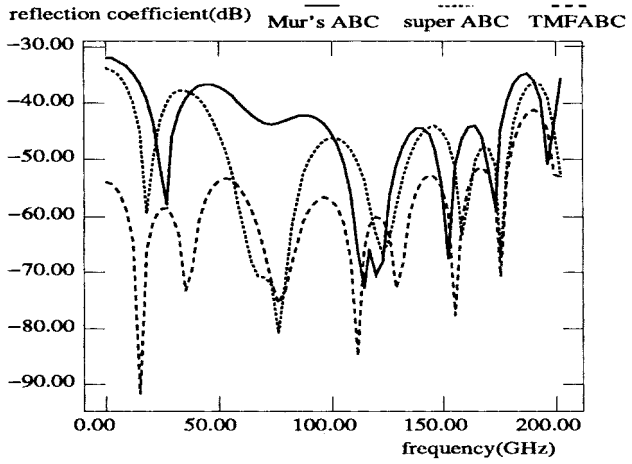

Fig. 3. Reflection coefficients of three ABC's as functions of the frequency in the example.

to uniquely decide the integral area in the power–current definition for voltage on different conductors. The last reason is that the linearity property of interconnects will be lost if one is using the power–current definition for voltage. From Fig. 2, we can see that no matter what value of the adjustable ϵ_{eff} is taken, TMFABC is more efficient than Mur's first-order ABC and its super algorithm. The reflection coefficient of Mur's first-order ABC with $\epsilon_{\text{eff}} = 10.0$, reflection coefficient of TMFABC with $\epsilon_{\text{eff}} = 6.0$, and reflection coefficient of super Mur's first-order ABC with $\epsilon_{\text{eff}} = 11.0$ as functions of frequency are shown in Fig. 3, from which it is easy to see that as expected in Section II, TMFABC improves Mur's first-order ABC more effectively at lower frequencies.

In order to illustrate that TMFABC is still efficient to absorb evanescent waves, we apply TMFABC on the side boundaries and the far-end boundary. On the top boundary, four ABC's (i.e., Mur's first-order ABC, TMFABC, super Mur's first-order ABC, and Mur's second-order ABC) have been used for comparison. The temporal E_x at $4\Delta x$ above the strip center and $z = 10\Delta z$ is given in Fig. 4, clearly showing that TMFABC is efficient to absorb evanescent waves, but the other three ABC's are not.

IV. Conclusion

In this paper, we developed a twofold Mur's first-order ABC by applying Mur's first-order ABC twice on two diagonally neighbor

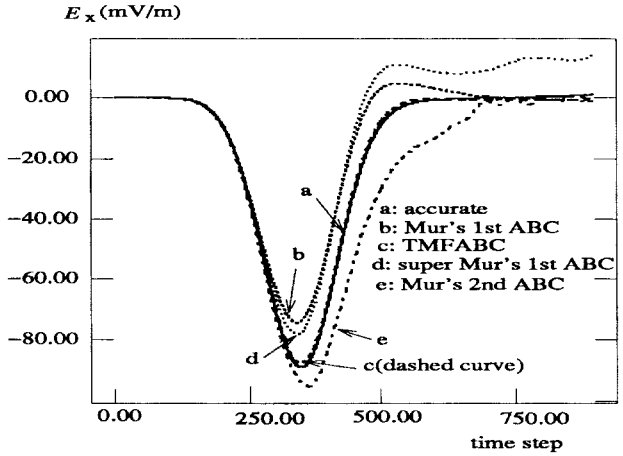

Fig. 4. Comparison of E_x from four ABC's applied on the top boundary with the accurate result in the example.

nodes on the x–t, y–t, and z–t planes. The reflection error due to Mur's first-order ABC is canceled in TMFABC, thus the absorption efficiency is greatly improved. TMFABC is efficient for absorbing not only propagative waves, but also evanescent waves, and is very convenient for implementation to multilayered structures. It improves Mur's first-order ABC more effectively at lower frequencies. The performance improvement of TMFABC over some existing ABC's has been highlighted by an application example.

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